On the Progression of Situation Calculus Basic Action Theories: Resolving a 10-year-old Conjecture

Stavros Vassos and Hector J. Levesque

Department of Computer Science
University of Toronto

ACAC 2009 (AAAI 2008)

Overview

- Introduction
- The situation calculus
  - The basic action theories
  - The problem of progression
- Two results about the first-order definability of progression
- Conclusions

Introduction

The problem we examine lies in the field of *knowledge representation* and *reasoning about action and change*.

Given a *logical formalism* that is able to:

1. represent the current state of the world;
2. represent the dynamics of the world;
3. answer queries about the current state and the possible future states of the world,

we want to *update* the representation of the current state after action execution.

Introduction

The problem we examine lies in the field of *knowledge representation* and *reasoning about action and change*.

Given a *logical formalism* that is able to:

1. represent the current state of the world;
2. represent the dynamics of the world;
3. answer queries about the current state and the possible future states of the world,

we want to *update* the representation of the current state after action execution.

Think of it as an “advanced database system” based on some expressive logical language.

The situation calculus language

The *situation calculus* is a first-order predicate language with limited second-order* features:

- *Fluents* are like normal predicates but also depend on a situation argument, e.g.,

$$F(x, S_0).$$

The situation calculus language

The *situation calculus* is a first-order predicate language with limited second-order* features:

- *Fluents* are like normal predicates but also depend on a situation argument, e.g.,

$$F(x, S_0).$$

- S_0 is the initial situation.
- $do(A, S_0)$ is the resulting situation after action A has been performed in S_0.

The situation calculus language

The *situation calculus* is a first-order predicate language with limited second-order* features:

- *Fluents* are like normal predicates but also depend on a situation argument, e.g.,

$$F(x, S_0).$$

- S_0 is the initial situation.
- $do(A, S_0)$ is the resulting situation after action A has been performed in S_0.
- Situations are used to refer to future states of the world:

$$F(x, do(A, S_0)).$$

The basic action theories

A *basic action theory* $\mathcal{D}$ is a set of situation calculus sentences:

- KB: axioms that characterize the initial situation;
- DYN: axioms that represent how the world changes;
- FND: axioms* that define the space of situations.

The basic action theories

A *basic action theory* $\mathcal{D}$ is a set of situation calculus sentences:

- KB: axioms that characterize the initial situation; S_0
- DYN: axioms that represent how the world changes; $s \rightsquigarrow do(a, s)$
- FND: axioms* that define the space of situations.

The basic action theories

A *basic action theory* $\mathcal{D}$ is a set of situation calculus sentences:

- KB: axioms that characterize the initial situation; S_0
- DYN: axioms that represent how the world changes; $s \rightsquigarrow do(a, s)$
- FND: axioms* that define the space of situations.

1 Represent the current state of the world: KB

2 Represent the dynamics of the world: DYN

3 Answer queries about the future based on *entailment*:

- $\mathcal{D} \models \neg F(c, S_0)$
- $\mathcal{D} \models F(c, do(A, S_0))$
- $\mathcal{D} \models \forall s(do(A, S_0) \sqsubseteq s \supset F(c, s))$

Problem: basic action theory progression

A *basic action theory* $\mathcal{D}$ is a set of situation calculus sentences:

- KB: axioms that characterize the initial situation;
- DYN: axioms that represent how the world changes;
- FND: axioms* that define the space of situations.

S_0	$do(a, S_0)$	future of $do(a, S_0)$
KB	KB$\cup$DYN$\cup$FND	KB$\cup$DYN$\cup$FND

Problem: basic action theory progression

A *basic action theory* $\mathcal{D}$ is a set of situation calculus sentences:

- KB: axioms that characterize the initial situation;
- DYN: axioms that represent how the world changes;
- FND: axioms* that define the space of situations.

S_0	$do(a, S_0)$	future of $do(a, S_0)$
KB	KB∪DYN∪FND	KB∪DYN∪FND
–	KB′	KB′∪DYN∪FND

Problem: basic action theory progression

A *basic action theory* $\mathcal{D}$ is a set of situation calculus sentences:

- KB: axioms that characterize the initial situation;
- DYN: axioms that represent how the world changes;
- FND: axioms* that define the space of situations.

S_0	$do(a, S_0)$	future of $do(a, S_0)$
KB	KB∪DYN∪FND	KB∪DYN∪FND
–	KB′	KB′∪DYN∪FND

Correct progression wrt a: replace KB by a KB′ such that:

- KB′, $\mathcal{D}$ entail the same first-order sentences about $do(a, S_0)$;
- KB′∪DYN∪FND and $\mathcal{D}$ entail the same first-order sentences about the future of $do(a, S_0)$.

Two definitions of progression

S_0	$do(a, S_0)$	future of $do(a, S_0)$
KB	KB∪DYN∪FND	KB∪DYN∪FND
–	KB′	KB′∪DYN∪FND

- *LR*-progression [Lin and Reiter 1997]:
  - model-theoretic specification of KB′;
  - always correct;
  - comes with a strong negative result: there are theories for which there is no first-order representation of KB′.
- *FO*-progression [Pednault 1987]:
  - the specification of KB′ is based on first-order entailments;
  - open whether it is always correct or not;
    Lin and Reiter [1997] conjectured that FO-progression is too weak!

Two definitions of progression

S_0	$do(a, S_0)$	future of $do(a, S_0)$
KB	KB∪DYN∪FND	KB∪DYN∪FND
–	KB′	KB′∪DYN∪FND

- *LR*-progression [Lin and Reiter 1997]:

  KB′ second-order but always correct

- *FO*-progression [Pednault 1987]:

  KB′ first-order but Lin & Reiter conjectured it is incorrect

Two definitions of progression

S_0	$do(a, S_0)$	future of $do(a, S_0)$
KB	KB∪DYN∪FND	KB∪DYN∪FND
–	KB′	KB′∪DYN∪FND

- *LR*-progression [Lin and Reiter 1997]:
  KB′ second-order but always correct
- *FO*-progression [Pednault 1987]:
  KB′ first-order but Lin & Reiter conjectured it is incorrect

This paper:

- We prove the conjecture by Lin and Reiter and show that *FO*-progression is indeed incorrect in the general case.

Result 1: *FO*-progression is too weak in general

- Consider the simple sit-calc language that consists of:
  - the fluent $F(x, s)$;
  - the actions A, B;
  - the function $n(x)$;
  - the constant 0.

- We specify a basic action theory KB∪DYN∪FND and a sentence ϕ about the future of $do(A, S_0)$ such that:

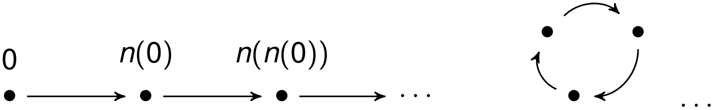
$$\text{KB}\cup\text{DYN}\cup\text{FND}\models \phi \quad \text{but} \quad \text{KB}'\cup\text{DYN}\cup\text{FND}\not\models \phi,$$

  where KB′ is a *FO*-progression of KB wrt *A*.

- KB, DYN, and ϕ exploit the weaknesses of first-order logic wrt formalizing true arithmetic.

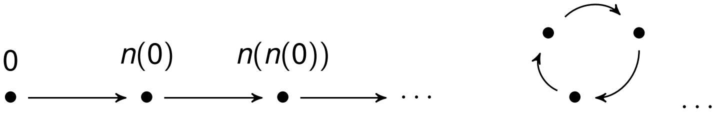
Result 1: *FO*-progression is too weak in general

- KB consists of the following four sentences:
  - $\forall x (x \neq 0 \equiv \exists y \; n(y) = x)$
  - $\forall x \forall y (n(x) = n(y) \supset x = y)$
  - $F(0, S_0) \wedge \forall x \, (F(x, S_0) \supset F(n(x), S_0))$
  - $\exists x \; \neg F(x, S_0)$



Result 1: *FO*-progression is too weak in general

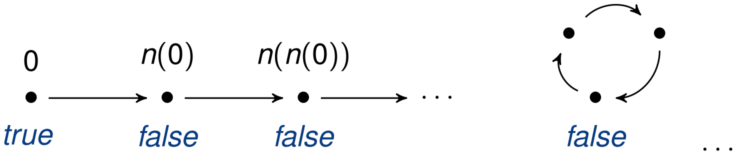
- KB consists of the following four sentences:
  - $\forall x (x \neq 0 \equiv \exists y\ n(y) = x)$
  - $\forall x \forall y (n(x) = n(y) \supset x = y)$
  - $F(0, S_0) \wedge \forall x\, (F(x, S_0) \supset F(n(x), S_0))$
  - $\exists x\ \neg F(x, S_0)$



- *In all models of KB there is always some object that is not reachable from* 0.

Result 1: *FO*-progression is too weak in general

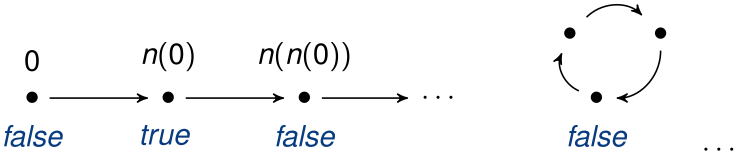
- DYN consists of the following sentence:
  - $$F(x, do(a,s)) \equiv \; a = A \wedge x = 0 \vee$$
    $$a = B \wedge \neg F(x,s) \wedge \exists y (x = n(y) \wedge F(y,s))$$



- Action A makes $F(x,s)$ false for all x except for 0.

Result 1: *FO*-progression is too weak in general

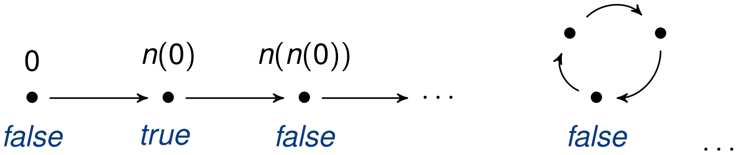
- DYN consists of the following sentence:
  - $$F(x, do(a,s)) \equiv \; a = A \wedge x = 0 \vee$$
    $$a = B \wedge \neg F(x,s) \wedge \exists y (x = n(y) \wedge F(y,s))$$



- Action B after A makes $F(x,s)$ false for all x except for $n(0)$.

Result 1: *FO*-progression is too weak in general

- DYN consists of the following sentence:
  - $$F(x, do(a, s)) \equiv \; a = A \wedge x = 0 \vee \\ a = B \wedge \neg F(x, s) \wedge \exists y (x = n(y) \wedge F(y, s))$$



- Action B after A makes $F(x, s)$ false for all x except for $n(0)$.
- *The objects that are unreachable from* 0 *can never become true after action A*.

Result 1: *FO*-progression is too weak in general

- In all models of KB∪DYN∪FND there is always some object that is not reachable from 0.
- In all models of KB∪DYN∪FND the objects that are not reachable from 0 can never become true after action A.
- Let ϕ be the sentence $\exists x \forall s (do(A, S_0) \sqsubseteq s \supset \neg F(x, s))$.

It follows that KB∪DYN∪FND$\models \phi$.

Result 1: *FO*-progression is too weak in general

- In all models of KB∪DYN∪FND there is always some object that is not reachable from 0.
- In all models of KB∪DYN∪FND the objects that are not reachable from 0 can never become true after action *A*.
- Let ϕ be the sentence $\exists x \forall s (do(A, S_0) \sqsubseteq s \supset \neg F(x, s))$.

It follows that KB∪DYN∪FND$\models \phi$.

- Let KB′ be $\forall x \big(F(x, do(A, S_0)) \equiv x = 0 \big)$.

Result 1: *FO*-progression is too weak in general

- In all models of KB∪DYN∪FND there is always some object that is not reachable from 0.
- In all models of KB∪DYN∪FND the objects that are not reachable from 0 can never become true after action *A*.
- Let ϕ be the sentence $\exists x \forall s (do(A, S_0) \sqsubseteq s \supset \neg F(x, s))$.

It follows that KB∪DYN∪FND$\models \phi$.

- Let KB′ be $\forall x \big(F(x, do(A, S_0)) \equiv x = 0\big)$.
- KB′∪DYN∪FND has a model where all objects are reachable from 0.
- In this model every object may become true after action *A* by a series of *B* actions.

It follows that KB′∪DYN∪FND$\not\models \phi$.

Implications of Result 1

There is no general definition for a correct progression KB′ that will work within first-order logic in all cases.

Three alternatives:

- limit the type of sentences about the future of $do(a, S_0)$:
  - e.g. consider queries about a *specific situation only*: [Lin and Reiter 1997], [Shirazi and Amir 2005].
- limit the type of the action theories:
  - e.g. consider theories with *local effects*: [Thielscher 1999], [Liu and Levesque 2005], [Vassos, Lakemeyer, and Levesque 2008].
- weaken the form of progression:
  - e.g. consider a progression that is *sound but not complete*: [Liu and Levesque 2005].

Implications of Result 1

There is no general definition for a correct progression KB′ that will work within first-order logic in all cases.

Three alternatives:

- limit the type of sentences about the future of $do(a, S_0)$:
  - e.g. consider queries about a *specific situation only*: [Lin and Reiter 1997], [Shirazi and Amir 2005]. Result 2
- limit the type of the action theories:
  - e.g. consider theories with *local effects*: [Thielscher 1999], [Liu and Levesque 2005], [Vassos, Lakemeyer, and Levesque 2008].
- weaken the form of progression:
  - e.g. consider a progression that is *sound but not complete*: [Liu and Levesque 2005].

Result 2: *FO*-progression is sometimes correct

- *FO*-progression is *correct* for any sentence that just talks about a *specific situation* such as S_1, S_2 [Lin and Reiter 1997].
- Result 1 shows that *FO*-progression is *not correct* when *unrestricted quantification* over future *situations* is allowed ($\exists x \forall s\ \Phi(x, s)$).

Result 2: *FO*-progression is sometimes correct

- *FO*-progression is *correct* for any sentence that just talks about a *specific situation* such as S_1, S_2 [Lin and Reiter 1997].
- Result 1 shows that *FO*-progression is *not correct* when *unrestricted quantification* over future *situations* is allowed ($\exists x \forall s\, \Phi(x, s)$).
- However, we were able to show that *FO*-progression is indeed *correct* for a practical class of sentences that allows some quantification over situations, such as:
  - invariants of the form $\forall s\, \Phi(s)$, *"in all future situations Φ holds"*;
  - sentences of the form $\exists s\, \Phi(s)$, *"there is a future situation where Φ holds"*.

Conclusions

- We investigated the problem of *progressing* of a basic action theory in the *situation calculus*.
- We proved *two major results*: first, one that justifies the second-order definition of progression by Lin and Reiter, and second, one that shows that under conditions the simpler first-order definition is adequate.
- The first result consists a proof for a problem that has been open since it was first formalized in [Lin and Reiter 1997].
- We conclude that both results have a *positive flavor*:
  - Result 1: it is tricky to find an example theory and query where *FO*-progression is too weak;
  - Result 2: *FO*-progression is always strong enough for a broad class of queries.